The Islamic university of Gaza
Department of Mathematics
Date : 6/1/2007

Calculus A (MathA1401)
Final exam
Time : 2 hours

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الاسم:	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total
الرقم الجامعي:											
اسم المدرس:	6	6	10	10	15	10	5	9	9	20	100

[Q1] The graph of the function $y = \frac{1}{x^2}$ is shifted 2 units to the right and 3 units down.
Find the equation of the new graph.

[Q2] Find the positive constant k for which $y = k\sqrt{5x+1}$ satisfies $y\frac{dy}{dx} = 1$.

[Q3] Find $\dfrac{dy}{dx}$ if

(a) $y = \dfrac{\sin x^{2}}{1+\cos^{2} x}$

(b) $y = \displaystyle\int_{1}^{2\sec x} \frac{dt}{\sqrt{t^{2}-4}}$, $\quad 0 < x < \dfrac{\pi}{2}$ $\quad$ (Simplify your answer)

[Q4] Evaluate the following limits:

(a) $\lim_{t \to -2} \frac{t^2 - t - 6}{t^2 + 5t + 6}$

(b) $\lim_{x \to \frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$

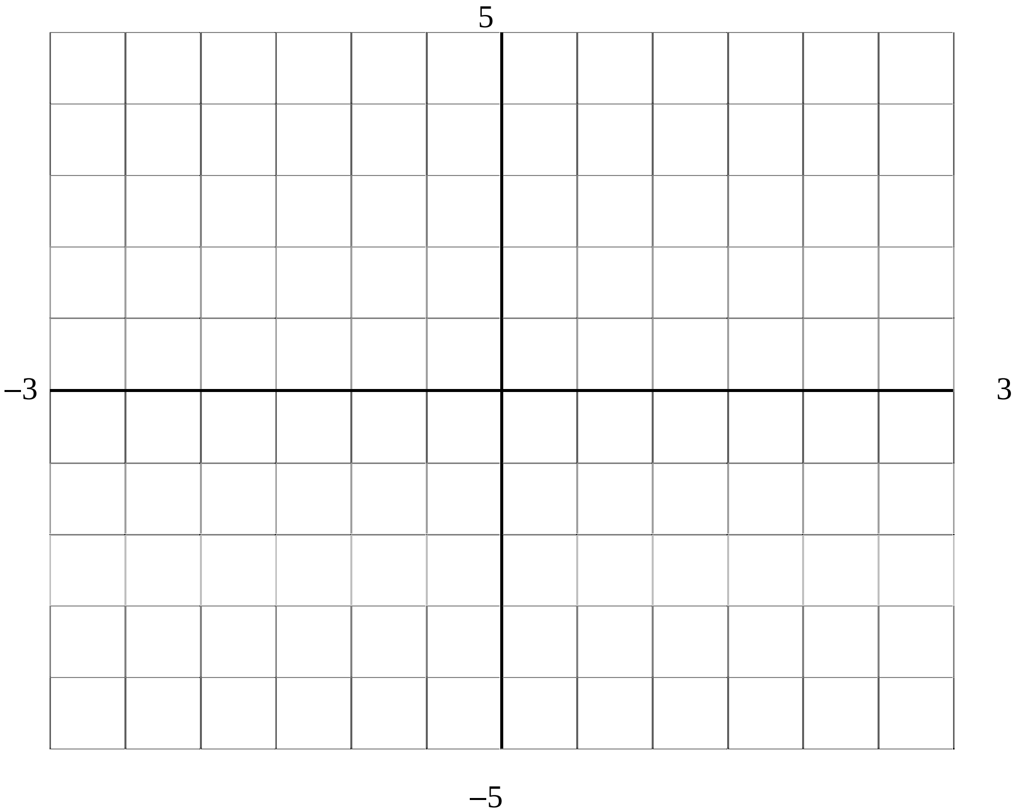
[Q5] Let $f(x)=\dfrac{x^2-2}{x^2-1}$ be a function with $f'(x)=\dfrac{2x}{(x^2-1)^2}$ and $f''(x)=\dfrac{-(6x^2+2)}{(x^2-1)^3}$

(a) Find all asymptotes for the graph of $f(x)$.

(b) Classify the critical points of $f(x)$ as " local maximum", "local minimum", or other.

(c) List the intervals on which $f(x)$ is increasing, decreasing, concave up or concave down.

(d) Sketch the graph of $f(x)$ which lies in the rectangle $-3 \le x \le 3$ and $-5 \le y \le 5$.



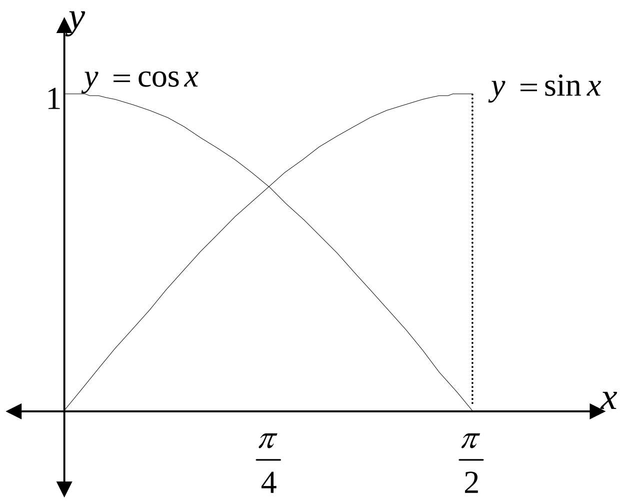
[Q6] Evaluate the following integrals:

(a) $\int (2x-1)(3+x)^{10}dx$

(b) $\int_0^1 |2x-1|\, dx$

[Q7] Suppose that the continuous function $f(x)$ has the property that $\int_{-1}^{5} f(x)dx = 13$ and $\int_{3}^{5} f(x)dx = -3$. Find $\int_{3}^{-1} \left(2f(x)+3\right) dx$.

[Q8] Find the area of the region bounded by the curves $y = \sin x$ and $y = \cos x$ from $x = 0$ to $x = \dfrac{\pi}{2}$ $\big($ The region is graphed below $\big)$



[Q9] <u>**Solve one and only one**</u> of the following two questions:

(a) Find the volume of the solid generated by revolving the region enclosed by the curves $y = x^2 + 1$, $y = 2$ about the line $y = 2$.

(b) Find the area of the surface generated by revolving the curve $y = \sqrt{x}$, $\frac{3}{4} \le x \le \frac{15}{4}$ about the x-axis.

[Q10] Circle the correct answer.

1) If $f(x) = \tan x$ then:

a) $\lim_{x \to \frac{\pi}{2}} f(x)$ exists.

b) $f(\frac{\pi}{2})$ exists

c) $f(x)$ is continuous at $x = \frac{\pi}{2}$

d) none of the above is true.

2) $\lim_{x \to 3^-} \frac{5}{(x-3)^3}$ equals

a) 5

b) ∞

c) $-\infty$

d) 0

3) If $f(x) = \int_1^{2\sin x} \sqrt{1-t^2}\, dt$ **,** then $f(\frac{\pi}{6}) =$

a) $\sqrt{1-4\sin^2 x}$

b) 2

c) 1

d) 0

4) $\lfloor -7.8 \rfloor$ equals

a) $-\lfloor 7.8 \rfloor$

b) -78

c) $-\lfloor 8.7 \rfloor$

d) non of the above

5) If $f(x)$ is differentiable on $[a,b]$, then $f(x)$ satisfies

a)The hypotheses of Rolle's theorem.

b) The hypotheses of the mean value theorem

c) The existence of $f''(x)\ \forall x \in [a,b]$

d) all of the above

6) If $f(x) = x + |x|$ and $g(x) = -x^2$, then $(f \circ g)(x)$ equals

a) $-2f(x)$

b) $-2g(x)$

c) $2g(x)$

d) 0

7) If $f(x)$ is increasing function and $g(x)$ is decreasing function then $(f-g)(x)$

a) is increasing

b) is decreasing

c) may be increasing or may be decreasing

d) non of the above

8) If $f(x)$ has a local maximum at an interior point $x = c$, then

a) $f(x)$ has an absolute maximum at $x = c$

b) $f(x)$ has an inflection point at $x = c$

c) $f'(x)$ exists

d) $f(x)$ has a critical point at $x = c$

9) If $\lim_{x \to c} f(x)$ exists, then

a) $f(c)$ exists

b) $f(x)$ is continuous at $x = c$

c) $f(x)$ is a constant function

d) $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$

10) If $f'(c)$ exists, then at $x = c$

a) $f(x)$ has a vertical tangent

b) $f(x)$ has a horizontal tangent

c) $f(x)$ is continuous

d) non of the above.